Finite Element Analysis of Nonlinear Flow Problems of Nanofluids

Synopsis of the Ph.D. Thesis

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ABSTRACT

The study of heat and mass transport of nanofluids is of great practical importance due to its impact on many branches of science and engineering. Nanofluids are novel fluids characterized by the presence of nano-sized particles (1-100 nm) dispersed within a base fluid. Distinctive characteristics of nanofluids make them potentially useful in various applications in heat and mass transfer. Nanoparticles are of great scientific interest as they effectively act as bridge between atomic or molecular structures and bulk materials. Many fluids are available in real life on the basis of relationship between shear stress and rate of strain, which are classified as Newtonian and non-Newtonian. Nanofluid flows has been the field of very active research for the last few decades. Thus, it is important to study Newtonian and non-Newtonian nanofluid flow characteristics as they have wide applications in engineering, biomedicines and industrial processes.

In the present research work, the numerical study of two-dimensional, steady, incompressible, laminar boundary layer flow problems of nanofluid in the presence of magnetic field, thermal radiation and chemical reaction under different boundary conditions in various geometries such as flat plate, non-linear stretching sheet, wedge and channel has been undertaken. The underlying governing equations for these problems are nonlinear differential equations which often have no solution or it is extremely difficult to obtain the solution analytically. Therefore, to solve such problems, finite element method (FEM) is utilized employing linear, quadratic and Hermite interpolation polynomials.

The governing equations for the nanofluid flow problems have been solved numerically by using similarity transformations which are then transformed into a system of nonlinear ordinary differential equations. To ensure the accuracy of the current work, it has been validated against previous studies for specific cases of the problem. The influence of the different physical parameters on velocity, microrotation, temperature and concentration functions has been elucidated in detail. Further, the numerical results for the rate of heat and mass transfer, skin friction coefficient and wall couple stress have also been computed. The findings are presented through graphs and tables highlighting their engineering significance. It is expected that the collective findings may offer a nuanced understanding of nanofluid behaviors, with practical implications for designing heat transfer systems that employ nanofluids as their principal heat

exchange medium, and support applications in futuristic technology which thrives on heat enhancement research like renewable and geothermal energy arena, microfluidic systems, nano drug delivery and cancer treatment.

Chapter 1
INTRODUCTION

In recent years, the study of nanofluids has garnered significant attention as a frontier in thermal and fluid dynamics research owing to their exceptional thermal conductivity and rheological properties. Nanofluids, consisting of nanoparticles dispersed in conventional base fluids, exhibit promising heat transfer characteristics that have the potential to revolutionize various engineering applications. However, the research landscape has expanded beyond traditional nanofluids, with contemporary investigations increasingly focusing on the study of hybrid nanofluids and micropolar nanofluids, reflecting a growing recognition of the need for more tailored and versatile fluid formulations.

The term hybrid nanofluids refers to nanofluids that combine a mixture of different types of nanoparticles to enhance specific properties. This approach allows researchers to synergistically achieve the unique features of multiple nanoparticle types, opening avenues for customizing nanofluid characteristics to meet specific application requirements. Simultaneously, the investigation of micropolar nanofluids has emerged as a cutting-edge field within nanofluid research. Micropolar nanofluids combine the characteristics of micropolar fluids, which account for microscale effects such as microrotation and microinertia, with the enhanced thermal properties conferred by nanoparticle dispersion. This interdisciplinary approach adds a new dimension to the study of nanofluid dynamics, offering a more comprehensive understanding of the fluid behavior and interactions within the microstructure of the fluid.

While the study of nanofluids has progressed considerably, the exploration of micropolar nanofluids is gaining importance due to their potential to address specific limitations and enhance overall performance in diverse applications. However, despite the growing interest, critical research gaps exist in understanding the fluid flow dynamics of these advanced nanofluid formulations, especially over different geometries, considering the collective effects of nanoparticles volume fraction, thermal radiation, magnetic field and chemical reaction while accounting for convective boundary condition. Notably, there is a paucity of comprehensive investigations employing finite element method solutions for these complex problems. The current literature often lacks a thorough investigation of finite element analysis to study these phenomena,

limiting the depth of understanding and hindering the applicability of findings to real world engineering systems.

The study aims to formulate a mathematical model and employ the finite element method to obtain numerical solutions for non-linear boundary layer flow problems involving nanofluids over diverse geometries. The investigation takes into account various parameters and boundary conditions to comprehensively analyze the fluid dynamics in these complex scenarios.

1.1 RESEARCH OBJECTIVES

The problems in nanofluids are challenging as one has to deal with coupled non-linear partial differential equations. To address these challenges, the following research objectives are formulated:

1. Develop a mathematical model for non-linear boundary layer flow problems of nanofluids over distinct geometrical surfaces.
2. Obtain numerical solution for these non-linear coupled partial differential equations using finite element method by setting a computational algorithm.
3. Analyze the influence of different governing parameters that affect the fluid flow.
4. Identify the key parameters that significantly influence the skin friction coefficient and rate of heat and mass transfer.

Chapter 2
LITERATURE REVIEW

Over the past decade, nanotechnology has emerged as a focal point in various engineering domains, particularly in the realm of fluid cooling and heating. Choi and Eastman [1] introduced the term "nanofluid" to describe fluids infused with nanoparticles ranging from 1 to 100 nanometers in size. Initial research by Masuda et al. [2] emphasized thermal conductivity enhancement using ultra-fine particles, with subsequent studies revealing that maintaining a minimum of 5% nanoparticle volume fraction enhances the thermal characteristics of nanofluids [3]. The applications of nanofluids extend across diverse fields such as power, transportation, manufacturing and electronic devices. Beyond traditional engineering applications, nanofluids demonstrate promise in solar devices [4], combustion processes [5] and space technology [6]. Assael et al. [7] outlined numerous potential applications for nanofluids in heat transfer. Moreover, nanofluid flows play a crucial role in biomedical applications including nano-drug delivery, cancer treatment, tumor therapy and the dynamics of physiological fluids [8]. Recently, Nanofluid models have facilitated the mathematical analysis of human blood flow offering a quicker and cost-effective approach [9].

The micropolar fluid theory offers a robust framework for comprehensively understanding and modeling the microstructural properties of fluids in diverse applications, ranging from biomechanics, slurry technologies to chemical and material engineering processes. Eringen [10] formulated the theory of micropolar fluid to model such effects and accurately imitate blood, synovial fluids (i.e biological fluid), smart fluids and various colloidal suspensions. Swapna et al. [11] studied heat and mass transfer of micropolar fluid flow over a permeable linearly stretched cylinder using finite element method. Islam et al. [12] carried out computational analysis on MHD micropolar fluid flow over a flat plate using finite difference technique. Recently, Usafzai and Aly [13] derived the exact solutions for temperature functions considering Hiemenz micropolar fluid flow.

The micropolar nanofluid model emerges as a refined approach to overcome the limitations of traditional nanofluid model, particularly in explaining fluids with unique microscopic characteristics driven by local particle structures and microrotation. Addressing this, Bourantas

and Loukopoulos [14] investigated impact of magnetic field on the flow of micropolar Al2O3-water-based nanofluid by employing the moving least squares method. Similarly, Turk and Tezer-Sezgin [15] utilized the FEM to explore micropolar nanofluid flow. Further, Rafique et al. [16] delved into micropolar nanofluid models considering inclined surface. Nisar et al. [17] analyzed the impact of radiation on Au-blood flow using the three-stage Lobatto technique. Further, Rajkumar and Reddy [18] explored the influence of heat sink/source and thermal radiation on magnetohydrodynamic Au-blood nanofluid, emphasizing its relevance in radiotherapy and drug delivery models.

In industrial applications such as wire drawing, hot rolling and chemical coating of vertical flat plates, the boundary layer flows past flat bodies are regularly encountered [19]. Siegal [20] conducted the initial investigations over vertical plate. Aziz and Khan [21] investigated convectively heated vertical plate by considering Brownian motion and thermophoresis effects, employing fourth order Runge-Kutta-Fehlberg (RKF) method. Later, Mustafa et al. [22] examined nanofluid flow over a vertical plate considering the influence of non-linear radiative heat flux. Othman and Mahdy [23] studied the free convection boundary layer flow over a vertical plate using semi-analytical shifted Legendre collection method. Jha and Samaila [24] numerically investigated impact of thermal radiation on the flow over a flat plate with convective boundary condition. Klazly and Bognar [25] studied the nanofluid flow over flat plate using CFD simulations. Recently, Khan et al. [26] presented the study based on entropy generation for Blasius flow.

Flows over the stretching surface have significant impact in fluid dynamics. The stretching sheet is used in numerous practical operations including manufacturing of paper and rubber sheets, drawing of plastic and wire, hot rolling and glass blowing [27, 28]. Studies reported in the literature primarily center around linear stretching sheet. But in actual practice a stretching sheet may not always be linear which is pointed out by Gupta and Gupta [29]. Later, Vajravelu [30] investigated heat and flow characteristics for viscous fluid over non-linear stretching sheet. Bataller [31] considered prescribed surface temperature and prescribed heat flux boundary conditions to study heat transfer on non-linear stretching sheet. Ferdows et al. [32] used RKF fourth–fifth order numerical method to investigate the effects of thermal radiation and convective surface boundary condition on free convective MHD flow over vertical stretching sheet. Analytical solution of a

quiescent viscous fluid over non-linear stretching surface is presented by Kazemi et al. [33]. Rasool et al. [34] applied fourth order Runge-Kutta (RK) scheme to study Casson nanofluid flow through porous medium over nonlinear stretching sheet. Recently, Rashmi et al. [35] investigated heat transfer for silver and iron oxide nanofluid over a linear stretching sheet by applying shooting approach. Very recently, Patil et al. [36] employed the fourth-order RK technique and shooting method to analyze the heat and mass transport in MHD Williamson nanofluid flow over a porous linear stretching surface with chemical reaction and thermal radiation.

The study of transport phenomena along a wedge holds particular significance from a technological perspective with numerous applications in aerodynamics, heat exchangers, chemical engineering and the containment of nuclear waste. Falkner and Skan [37] derived the fundamental Falkner–Skan equation to describe the laminar boundary layer flow past a wedge shaped body. Later, Hartree [38] obtained the numerical solution of this equation. Ishak et al. [39] obtained numerical solutions using the Keller-box method for boundary layer flow of non-Newtonian fluid over a moving wedge. Khan and Pop [40] examined the steady laminar flow of nanofluids past a stretching wedge considering Brownian motion and thermophoresis. Investigations into mixed convective nanofluid stagnation point flow over wedge was conducted by Ellahi et al. [41] while Ahmed et al. [42] examined viscous dissipation and radiative heat flux over a moving wedge utilizing volume fraction. The flow of radiative nanofluids over a wedge has also been considered by Amar and Kishan [43]. Recently, Kho et al. [44] studied hybrid nanofluid flow over a permeable wedge with viscous dissipation and thermal radiation.

Laminar flows in porous channel play a crucial role in various engineering applications including artificial dialysis, capillary and blood oxygenator flows, filter and porous pipe design, transpiration cooling, microfluidic devices and filtration processes. Berman [45] initiated the study of the two-dimensional laminar flow of a Newtonian fluid in a porous channel subjected to uniform suction. Ziabakhsh and Domairry [46] employed homotopy analysis method to investigate effect of magnetic field on the laminar viscous flow within a semi-porous channel. Magnetohydrodynamics blood flow was simulated by Misra and Sinha [47] and concluded that magnetic field can reduce the volumetric blood flow rate. Hassan and Rashidi [48] applied the homotopy method to obtain the semi-analytical solution flow of a micropolar fluid through a porous channel with mass injection. Noreen et al. [49] investigated the impact of Cu suspensions

for blood flow in vertically aligned channel. The hydromagnetic micropolar nanofluid over a vertical channel has been examined by Sarkar and Kundu [50] using RK fourth order method with heterogeneous heat source and sink. Alsagri et al. [51] utilized carbon nanotubes over stretchable upright cylinder to study the features of electrically conducting finite film flow of human blood. Elelamy and Elgazery [52] investigated bacterial growth in the heart valve by developing a mathematical model where nanoparticles are used for simulating antibody properties and antibacterial activities. Further, Ahmad et al. [53] applied thermal radiation and employed quasi-linearization method to examine the micropolar flow within channel walls. Nisar et al. [54] analyzed the influence of chemical reaction on magnetohydrodynamic micropolar flow through permeable walls. Recently, Jalili et al. [55] provided the solution to micropolar fluid transport phenomena through permeable channel using semi-analytical HAN and modified AGM methods.

Chapter 3

HEAT TRANSFER NANOFLUID FLOW ACROSS A PERMEABLE FLAT PLATE

The mathematical model delves into nanofluid flow over a permeable flat plate under convective boundary condition. The fundamental equations get transformed into a non-linear form by choosing appropriate similarity transformations and are solved numerically by substantiated novel FEM code by employing linear and quadratic interpolation functions. The study showcases the reliability of the FEM code through numerical analysis, revealing noteworthy trends in temperature and velocity profiles. The effect of permeability parameter and convective parameter on velocity and temperature functions are presented graphically and elucidated in detail. The numerical results for the rate of heat transfer and skin friction coefficient are reported in tabular form for selected parameters. The study adds to engineering science literature of computational multiphysical nanofluid dynamics simulation.

Chapter 4

ENHANCED HEAT TRANSFER USING ENGINE OIL NANOFLUID FLOW PAST A VERTICAL FLAT PLATE

This study explores the heat transfer properties of five different nanofluid flow past an oil cooling system. The numerical investigation is carried out by formulating nanofluid volume fraction in engine oil based nanofluids containing solid particles of Molybdenum disulfide (MoS_2), Aluminium oxide (Al_2O_3), Titanium oxide (TiO_2), Graphene oxide (GO), and Copper (Cu). The system of PDE are converted to ODE using appropriate similarity transformations and then solved by employing the FEM. The influence of different nanoparticles under study on temperature and velocity distributions are analysed graphically. Additionally, the impact of various nanoparticles, for the key parameters on the rate of heat transfer and coefficient of skin friction are presented in tabular form. The GO nanoparticles suspended in engine oil exhibits the lowest temperature and skin friction coefficient, and highest heat transfer capabilities. It may therefore be considered as the best lubricant for engines as it offers low friction and temperature, potentially extending the engine component's lifespan.

Chapter 5

MHD NANOFLUID FLOW ALONG POROUS NONLINEAR STRETCHING SURFACE

The investigation focuses on magnetohydrodynamic nanofluid flow across a permeable nonlinear stretching surface. Basic equations get transmuted to a nonlinear model by applying condensed version of Lie group transformations which are explored numerically using FEM code. Linear and quadratic interpolation functions are utilized in computations. Validation of the FEM code against existing data reinforces the precision of the model. The results obtained are depicted graphically to illustrate the impact of nanoparticle volume fraction, permeability parameter, magnetic parameter, convective boundary parameter on species concentration, temperature, and velocity functions. Additionally, Sherwood number, heat transfer rate and skin friction coefficient are also computed varying nanoparticle volume fraction and presented graphically for the flow regime. The findings hold practical significance for heat transfer systems utilizing nanofluids as their principal heat exchange medium and support applications in futuristic technology which thrives on heat enhancement research like renewable and geothermal energy arena.

Chapter 6

HEAT AND MASS TRANSFER FLOW OF MAGNETO-MICROPOLAR NANOFLUID OVER A WEDGE

The mathematical model examines radiative and chemically reactive magneto-micropolar nanofluid flow over a wedge, considering convective boundary condition. Employing similarity transformations found by Falkner and Skan [37], the nonlinear governing equations are analyzed using finite element method. Validation with existing literature corroborates the accuracy of the proposed model. Analysis of the results elucidate the impact of various parameters namely nanoparticle concentration, material parameter, wedge parameter, convective boundary parameter, radiation parameter, Schmidt number and chemical reaction parameter on the temperature, species concentration, microrotation and velocity profiles. Moreover, the numerical results for heat and mass transfer rate, couple stress and skin friction coefficient are computed. The observed enhancement in thermal conductivity with increased nanoparticle volume fraction holds potential applications in aerospace engineering, heat and cooling systems, enhanced oil recovery, chemical reaction engineering processes and solar collector energy system. This research thus contributes to the existing body of knowledge by offering an intricate understanding of the behaviour and manipulation of magneto-micropolar nanofluid flow.

Chapter 7
BLOOD GOLD MICROPOLAR NANOFLUID FLOW WITHIN A PERMEABLE CHANNEL

Hermite interpolation polynomials are employed and a numerical model is formulated which investigates electrically conducting micropolar nanofluid blood gold flow within a permeable channel. By applying Berman's [45] similarity transformation along with chemical reaction and thermal radiation, the resulting governing equations are transmuted to fourth-order nonlinear ordinary differential equation which are solved utilizing finite element method. The accuracy of the FEM code is verified by comparing the present computations with existing work and influence of driving parameters is studied via surface plots. The research findings offer insights pertinent to 3D nanotechnological printers, drug delivery devices and nano-cooling systems where laminar flows in channels arise.

Chapter 8
SUMMARY AND CONCLUSIONS

In the present research work, nanofluid dynamics is employed to study the steady, laminar, two dimensional, incompressible fluid flows in various geometries such as flat plate, stretching sheet, wedge and channel under various physical situations. Using similarity transformations, the governing system of partial differential equations are transformed into system of nonlinear ordinary differential equations. The finite element analysis is presented to obtain the numerical solution for the problems considered. The impact of thermophysical parameters on velocity, microrotation, temperature and concentration functions are shown graphically. Numerical results for the skin friction coefficient, couple stress, heat and mass transfer rate are also computed. Computations are compared with existing work and are found to be in excellent agreement. Additionally, mesh independence of the finite element computations is also achieved.

The principal findings of the work carried out in this research are:

Chapter 3: The results highlight that an increase in nanoparticle concentration amplifies thermal conductivity and the permeability of the plate significantly influences rate of heat transfer and skin friction coefficient. Also, an increase in the convective boundary parameter enhances temperature and heat transfer rate.

Chapter 4: The study reveals that among the studied nanofluids, GO-engine oil exhibits the lowest velocity profiles and skin friction coefficient, reducing drag and friction forces. Additionally, MoS_2-engine oil shows a higher temperature distribution within the boundary layer compared to other nanoparticles, suggesting potential enhancements in thermal management and machinery performance. Overall, GO-engine oil's altered boundary layer thickness improves heat transfer rates, offering superior thermal performance in cooling systems.

Chapter 5: The study reveals that surge in the nanoparticles enhances nanofluid thermal conductivity. Heat transfer rate, skin friction coefficient and Sherwood number are sturdily influenced by the permeability of the sheet. Further, a significant increase in the convective boundary parameter boosts temperature and heat transfer rate whereas higher values of the magnetic parameter retard nanofluid flow.

Chapter 6: It is observed that nanoparticles volume fraction enhances the thermal conductivity of magneto-micropolar nanofluid. The temperature is boosted by increasing the radiation and convective boundary parameter. The rate of mass transfer is enhanced by stronger measurements of the chemical reaction parameter and Schmidt number which is relevant for improving the efficiency of processes involving mass transfer. The deceleration of species concentration by manipulating the chemical reaction parameter and Schmidt number provides a tool for industries to optimize chemical processes, control pollutant dispersion and reduce environmental impact.

Chapter 7: The study emphasizes the role of the Hartmann number in controlling blood velocity. The chemical reaction parameter exerts a diminishing effect on species diffusion and rate of mass transfer within the flow. The results of the present investigation may be helpful in analyzing flow models related to drug-delivery systems as gold nanoparticles are proficient drug-delivery and drug-carrying medium.

Chapter 9
FUTURE SCOPE

The work carried out in the research provides a foundation for extending its applicability to diverse areas within nanofluid dynamics, suggesting potential applications in alternative flow geometries such as conical, cylindrical and spherical surfaces, while acknowledging the unexplored domains of unsteady and turbulent nanofluid flows and the prospect of three dimensional models. The scope can be broadened by addressing different heat and mass boundary conditions, including slip velocity, prescribed heat and mass flux and variable concentration. Future extensions may involve incorporating various nanomaterials with different base fluids like engine oil or propylene glycol to enhance heat and mass transfer outcomes. Moreover, the study hints at the vast potential for biomedical applications of nanofluids, specifically in cancer therapy and drug delivery, prompting further numerical simulations for exploring the capabilities of different nanoparticles as drug delivery vehicles in bio-fluids.

PUBLICATIONS

The following papers are published as a part of this research work:

- S. Gonsalves and G. Swapna, "FEM study of blood gold electrically conducting micropolar nanofluid flow within a permeable channel," *Numerical Heat Transfer, Part B: Fundamentals*, pp. 959-968, 2024. https://doi.org/10.1080/10407790.2024.2341436 (Scopus, Web of Science, SCI, Impact Factor: 1.1)
- S. Gonsalves and G. Swapna, "Convective Heat Transfer of Radiating Magneto-Micropolar Nanofluid Flow," *International Journal of Heat & Technology*, vol. 41, no. 4, pp. 959-968, 2023. https://doi.org/10.18280/ijht.410418
  (Scopus, Web of Science, ESCI, Impact Factor: 0.9)
- S. Gonsalves and G. Swapna, "Finite element study of nanofluid through porous nonlinear stretching surface under convective boundary conditions," *Materials Today: Proceedings*, 2023. https://doi.org/10.1016/j.matpr.2023.07.277 (Scopus)
- S. Gonsalves and G. Swapna, "Heat Transfer Analysis using Finite Element Method under Convective Boundary Condition," In *E3S Web of Conferences*, EDP Sciences, vol. 430, pp. 01248, 2023. https://doi.org/10.1051/e3sconf/202343001248 (Scopus)
- S. Gonsalves and G. Swapna, "Enhancing Convective Heat Transfer in Engine Oil: A Comparative Study of Various Nanoparticles," *Journal of Thermal Engineering*, communicated. (SCIE, Scopus, Web of Science, Impact Factor: 1.1)

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